

Lesson 2-4: Reasoning in Algebra

Geometric Arguments

If you want to argue a point, you need to make sure you have sufficient facts and data to back you up. The same is true in math. As I've said many times, one of the things we spend a lot of time doing in geometry is proving statements. In other words, you will be arguing a point.

What facts and data do we have at our disposal when arguing geometric proofs? There are four basic "facts and data" (or premises) that we base geometric proof on:

1. Definitions and undefined terms (points for instance)
2. Postulates of geometry
3. Previously accepted or proven geometric conjectures (theorems)

and

4. Properties of algebra, equality and congruence.

We have been looking at geometric definitions, terms, and postulates. Today we will review properties of algebra, equality and congruence.

Properties of equality

I'm sure you have seen many of these before. Perhaps you don't recognize them...I don't blame you. Unless you know why you need to know something, it can be hard to keep it in your head!

These properties of equality are extremely important. They allow us to move from one statement to another. Notice that they are all stated as conditionals. Conditional statements are the foundation of deductive reasoning. If we recognize a specific instance of the hypothesis, we can apply the conclusion. This is the core of doing a geometric proof.

Here they are:

Properties of Equality	
Addition Property	If $a = b$, then $a + c = b + c$
Subtraction Property	If $a = b$, then $a - c = b - c$
Multiplication Property	If $a = b$, then $a \cdot c = b \cdot c$
Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$
Reflexive Property	$a = a$
Symmetric Property	If $a = b$, then $b = a$
Transitive Property	If $a = b$ and $b = c$, then $a = c$
Substitution Property	If $a = b$, then b can be replaced by a in any expression

All I can say is ... learn these. OK, I can say one more thing. Learn these cold.

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Properties of algebra

You can assume any known property of algebra is true. However, here is one that you will be using a lot:

Properties of Algebra	
Distributive Property	$a(b + c) = ab + ac$

Solving equations = using deductive reasoning

Did you realize that every time you solve an algebraic equation, you are using deductive reasoning? You are. Every single time; you just aren't aware of it. We are going to become very aware of it. You will learn to justify every step you take while solving an equation. If you can get this down, you can handle geometric proofs. It is the exact same principle.

The trick is knowing what properties, definitions, postulates, etc. to apply in a given situation. It is extremely important that you have these down cold. I've told you to be compiling lists of these things: definitions, postulates and formulas. Keep these lists current. Review them regularly. Over time, as you use them, they will become familiar.

Some basic guidelines

If you are asked to solve an algebraic expression of equality, and justify each step (you **will** be asked to do this) here are some simple and basic guidelines. Basically, you need to think of yourself as a lawyer arguing a case in court. You need to prove your client innocent. To do so you need to convince the jury by justifying every point you make with facts and evidence. In mathematical proofs, our evidence is the definitions, properties, postulates, etc. that we have proven or taken to be true.

I know I'm repeating myself. I can't help it...it is absolutely critical you get this point.

YOU MUST HAVE YOUR TOOLS OF ARGUMENT DOWN PAT!!!

When you are considering how to justify a step in an algebra proof, consider what changed from the prior step:

1. If the changes are all on one side, you likely either simplified or used the substitution property or used the distributive property.
2. If the changes are on both sides of the equation, identify the operation performed (+, -, ×, ÷)...this will tell you which property of equality was used.

Example – Pg. 90, Example #1

Solve for x and justify each step.

Given: $m\angle AOC = 139$

$$\begin{array}{ll} m\angle AOB + m\angle BOC = m\angle AOC & \text{Angle Addition Postulate} \\ x + 2x + 10 = 139 & \text{Substitution Property (all on one side)} \\ 3x + 10 = 139 & \text{Simplify (all on one side)} \\ 3x = 129 & \text{Subtraction Property of Equality (-10 each side)} \end{array}$$

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$$x = 43$$

Division Property of Equality ($\div 3$ each side)

Example – Pg. 90, Check Understanding #1

Fill in each missing reason.

Given: \overline{LM} bisects $\angle KLN$

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Given

$$m\angle MLN = m\angle KLM$$

Definition of angle bisector

$$4x = 2x + 40$$

Substitution (**all on one side**)

$$2x = 40$$

Subtraction Property of Equality (**-2x each side**)

$$x = 20$$

Division Property of Equality ($\div 2$ each side)

Example – not in book

Solve for x and justify each step.

Given: $5x - 12 = 32 + x$

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Given

$$5x = 44 + x$$

Addition Property of Equality (**+12 each side**)

$$4x = 44$$

Subtraction Property of Equality (**-x each side**)

$$x = 11$$

Division Property of Equality ($\div 4$ each side)

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Properties of congruence

The reflexive, Symmetric and Transitive Properties of Equality have corresponding properties of congruence. You can use the properties of congruence to justify steps in the same way you can the properties of equality.

Properties of Congruence	
Reflexive Property	$\overline{AB} \cong \overline{AB}$ $\angle A \cong \angle A$
Symmetric Property	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$ If $\angle A \cong \angle B$, then $\angle B \cong \angle A$
Transitive Property	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$ If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$

Example – Pg 91, Check Understanding #3

Name the property of equality or congruence illustrated.

- a) $\overline{XY} \cong \overline{XY}$ **Reflexive Property of Congruence**
- b) If $m\angle A = 45$ and $45 = m\angle B$, then $m\angle A = m\angle B$ **Transitive Property of Congruence or Substitution Prop of Congruence**

Example – not in book

Name the property that justifies each statement.

- a) If $x = y$ and $y + 4 = 3x$, then $x + 4 = 3x$ **Substitution Prop of Equality**
- b) If $x + 4 = 3x$, then $4 = 2x$ **Subtraction Prop of Equality**
- c) If $\angle P \cong \angle Q$ and $\angle Q \cong \angle R$ and $\angle R \cong \angle S$, then $\angle P \cong \angle S$ **Transitive Prop of Congruence**

Assign homework

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